# Stringy instantons as strong dynamics

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ABSTRACT: We study the relation between stringy instantons and strong dynamics effects in type $II$B toric quiver gauge theories. By exploiting the involutive property of Seiberg duality we relate the classical constraint on the moduli space of the gauge theory with the stringy instanton contribution to the superpotential. The result holds for unitary, orthogonal and symplectic gauge groups.

## Contents

## 1. Introduction

Instantons are responsible for non-perturbative phenomena in $4D$ gauge theory and have, by now, found meaningful roles in string theory as well. Relevant work [1] has been done in different branches. It has been shown that instantons in string theory generate non perturbative contributions to the superpotential [2]–[12] and higher F-term contributions [13]. Moreover they have a role in model building, since they can be responsible for moduli stabilization [14] and for other phenomenological aspects like neutrino masses, supersymmetry breaking and gauge mediation [15]. Other results are found by adding fluxes [16] and more generally by looking at the string compactification scenarios [17].

One distinguishes between ordinary $D$-brane instantons and stringy $D$-brane instantons. Ordinary $D$-brane instantons are euclidean $D$-branes wrapping cycles in the geometry occupied by other space time filling $D$-branes. They reproduce ordinary instantons effects for the gauge theory living on the space time filling $D$-branes. Stringy $D$-brane instantons are euclidean branes wrapped over cycles in the geometry which are not occupied by any space-time filling brane.

In this note we investigate the relations between stringy instantons and strong dynamics effects in type $IIB$ toric quiver gauge theories. Stringy instanton contributions to the superpotential in quiver gauge theories have been shown to exist for SP(0), SU(1) and SO(3) nodes. The second and third cases are considered stringy since the low energy dynamics associated to those groups is trivial. The results, up to now, show that the stringy instanton contributions reproduce the non perturbative part of the superpotential of the gauge theory, i.e. part of the classical constraint on the moduli space. Here we present a clear-cut argument based on the involutive nature of the Seiberg duality, which explains the retrieval of the exact instanton contribution as a strong dynamical effect. We shall speak of equivalence or correspondence between the instantonic and gauge schemes.

The paper is organized as follows. First we give a general overview of the correspondence between stringy instantons and dynamical effects. Then, in section 2 we review the one-instanton action for a general quiver gauge theory and compute the contribution to the superpotential. In section 3 we argue that the correspondence is implied by the involutive property of Seiberg duality. In section 4 we give two examples, the $L^{121}$ and the $dP_1$ quiver gauge theories. In section 5 we discuss the correspondence for stringy instantons on SP(0) and SO(3) gauge groups in orientifolded quiver gauge theories, with clarifying examples. Finally we conclude in section 6. In appendix A we give some details to complete the analysis of section 2. In appendix B we compute the bosonic integration over the zero modes. In appendix C we review some known result in our interpretation.

## 1.1 Overview

Consider a quiver gauge theory with an SU(1) node and a tree level superpotential $W_{\text{tree}}$. A stringy instanton on a SU(1) node gives rise to a superpotential term [5, 8, 10]. The cycle wrapped by the euclidean $D$-brane is occupied also by one $D$-brane and the non trivial interaction lifts the fermionic zero modes. The resulting superpotential is

$$W = W_{\text{tree}} + W_{\text{inst}} \tag{1.1}$$

Gauge theories with a SU(1) gauge group are obtained as low energy (magnetic) descriptions of a strongly coupled SU($N_c$) gauge theory with $N_c + 1$ flavours. The low energy description of this strongly coupled SU($N_c$) gauge theory is a *limiting case* of Seiberg duality. Indeed, it can be described by a magnetic gauge group SU(1), where the elementary degrees of freedom are mesons and baryons. The baryons are the dual magnetic quarks. The classical moduli space of such a theory is not modified at quantum level. The classical constraint is imposed in the dual description by the addition of a non trivial superpotential for the mesons and the baryons, of the form

$$W_{\text{eff}} \sim B\mathcal{M}\tilde{B} - \det \mathcal{M} \tag{1.2}$$

We shall show that the second term in (1.2) is exactly reproduced by the stringy instanton contribution in (1.1). Here and in the rest of the paper we set to unity the dimension-full coefficients.

Relations between non perturbative dynamics and stringy instantons has been already observed in [4, 5] for cascading gauge theories. The correspondence we ascertain holds at every step of a cascade when the Seiberg duality is in the limiting case. The non perturbative contribution to the superpotential is then continuously mapped at every step until the bottom of the cascade [5].

## 2. Stringy instanton contribution

For the convenience of the reader we briefly review the basic instanton framework of relevance here. We describe the most general configuration with a SU(1) node in a toric quiver gauge theory and we place a stringy instanton on that node. We consider only rigid instantons, without adjoint fields charged under the SU(1) gauge group. The instantonic action for toric quiver gauge theories has been derived in [18]. Toric quiver gauge theories can be obtained by performing orbifold projection and higgsing of the $\mathcal{N} = 4$ theory. Along the same lines the instantonic action for toric quivers can be derived from the ADHM construction for $\mathcal{N} = 4$ theory.

The system consists of $N$ $D3$ branes and $k$ $D(-1)$ brane in type $II$ B. We refer to [1, 19] for reviews. The strings with endpoints attached to the $D3$ branes lead to SU($N$) $\mathcal{N} = 4$ SYM. The strings with endpoints attached to the $D(-1)$ branes lead to the neutral sector, uncharged under the gauge group. It includes bosonic moduli $a^\mu$ and fermionic zero modes $M^{\alpha\mathcal{A}}$ and $\lambda_{\dot{\alpha}\mathcal{A}}$ where $\alpha$ and $\dot{\alpha}$ denote the positive and negative chirality in four dimension and $\mathcal{A}$ is an SU(4) index (fundamental or anti fundamental) denoting the chirality in the transverse six dimensions. The equations of motion for the zero modes $\lambda_{\dot{\alpha}\mathcal{A}}$ implement the fermionic ADHM constraint. There is also a triplet of auxiliary bosonic fields $D^c$ whose equations of motion implement the bosonic ADHM constraint. The charged sector is associated with strings stretching between $D3$ branes and the $D(-1)$ branes. It includes bosonic spinors $\omega_{\dot{\alpha}}$ and $\bar{\omega}_{\dot{\alpha}}$ and fermions $\mu^{\mathcal{A}}$ and $\bar{\mu}^{\mathcal{A}}$. These fields are matrices of dimension $N \times k$.

In order to obtain the toric quiver gauge theory together with the instanton sector the whole field content has to be projected with the orbifold and then higgsed in a consistent way [18]. Notice that instanton moduli scale with the same Chan-Paton structure of ordinary gauge theory fields.

The resulting gauge theory is a toric quiver gauge theory with many gauge groups, where we can change the ranks of the groups by adding fractional $D$-branes. The instanton sector works in a similar way. There are $k$ instantons placed on each node, and we can add instantonic fractional branes (not to be confused with fractional instantons) to obtain a different numbers $k_i$ of instantons on the various nodes. Here we are interested in one instanton corrections without multi-instantons effects.

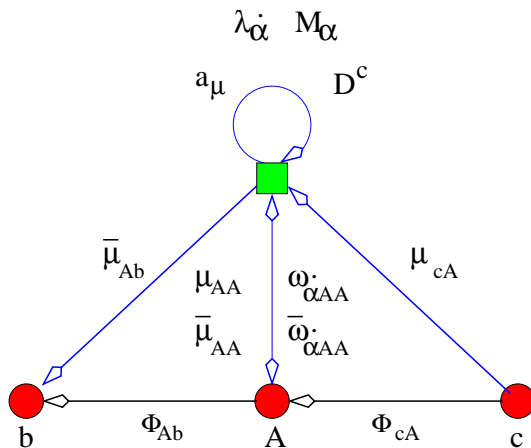From now on we consider one instanton placed on a SU(1) node in a generic toric quiver gauge theory (see figure 1).

**Figure 1:** Stringy instanton on a SU(1) node in a generic quiver. This is only the sector directly connected to the node $A$ of an anomaly free quiver.

| Sector | ADHM | Statistic | Chan-Paton |
|---------|------|-----------|------------|
| Charged | $\mu_{Ab}$ | Fermion | $k \times N_b$ |
| Charged | $\bar{\mu}_{cA}$ | Fermion | $N_c \times k$ |
| Charged | $\mu_{AA}$ | Fermion | $k \times N_A$ |
| Charged | $\bar{\mu}_{AA}$ | Fermion | $N_A \times k$ |
| Charged | $\omega_{\dot{\alpha}\,AA}$ | Boson | $k \times N_A$ |
| Charged | $\bar{\omega}_{\dot{\alpha}\,AA}$ | Boson | $N_A \times k$ |
| Neutral | $a_\mu$ | Boson | $k \times k$ |
| Neutral | $M_\alpha$ | Fermion | $k \times k$ |
| Neutral | $\lambda_{\dot{\alpha}}$ | Fermion | $k \times k$ |
| Neutral | $D^c$ | Boson | $k \times k$ |

**Table 1:** Spectrum of the ADHM moduli in the charged and in the neutral sector

We denote with $A$ the index associated with that node. The auxiliary instanton group is U(1). The node $A$ could be connected to the neighbor nodes with fields $\Phi_{Ab}$, for outgoing arrows in the quiver, or with fields $\Phi_{cA}$, for incoming arrows. In general, there could be more fields with the same gauge groups indexes. To simplify the notation we suppose here that every neighbor node is connected to the node $A$ with a single field. The general case is treated in the appendix A. The spectrum is reported in figure 1 and in table 1. The toric quiver represents the gauge sector. The neutral sector includes the bosonic zero mode $a_\mu$ and $D^c$, and the fermionic zero modes $M^\alpha$ and $\lambda^{\dot{\alpha}}$ (only the 4 component survive the orbifold projection in the one instanton case). There is a charged sector connecting the node $A$ and the instanton, given by $\omega_{\dot{\alpha}AA}, \bar{\omega}_{\dot{\alpha}AA}, \mu_{AA}, \bar{\mu}_{AA}$, and a charged sector connecting the instanton with the neighbor nodes, in a way similar to the field content of the gauge theory. For each existing outgoing arrow $\Phi_{Ab}$ there is a fermionic zero mode $\bar{\mu}_{Ab}$, and for each incoming arrow $\Phi_{cA}$ there is $\mu_{cA}$.

The instantonic action reads

$$S_{\text{inst}} = S_1 + S_2 + S_W \tag{2.1}$$

where

$$S_1 = i(\bar{\mu}_{AA}\omega_{\dot{\alpha}AA} + \bar{\omega}_{\dot{\alpha}AA}\mu_{AA})\lambda^{\dot{\alpha}} - iD^c(\bar{\omega}_{\dot{\alpha}AA}\tau^c\omega_{\dot{\alpha}AA}) \tag{2.2}$$

$$S_2 = \frac{1}{2}\sum_b \left[\bar{\omega}_{\dot{\alpha}AA}\Phi_{Ab}(\Phi_{Ab})^\dagger\omega_{\dot{\alpha}AA} + i\,\bar{\mu}_{Ab}(\Phi_{Ab})^\dagger\mu_{AA}\right] \tag{2.3}$$

$$+\frac{1}{2}\sum_c \left[\bar{\omega}_{\dot{\alpha}AA}(\Phi_{cA})^\dagger\Phi_{cA}\omega_{\dot{\alpha}AA} - i\,\bar{\mu}_{AA}(\Phi_{cA})^\dagger\mu_{cA}\right] \tag{2.4}$$

$$S_W = -\frac{i}{2}\sum_{b,c}\bar{\mu}_{Ab}\frac{\partial W}{\partial(\Phi_{cA}\Phi_{Ab})}\mu_{cA}, \tag{2.5}$$

Observe that the $S_W$ action involves derivatives of the superpotential with respect to bilinears of fields contracted on the $A$ index.[1]

The stringy instanton contribution is obtained by integrating over all the zero modes

$$Z = \mathcal{C}\int d\{a_\mu, M, \lambda^{\dot{\alpha}}, D, \omega_{AA}, \bar{\omega}_{AA}, \mu_{AA}, \bar{\mu}_{AA}, \bar{\mu}_{Ab}, \mu_{cA}\}\, e^{-S_{\text{inst}}} \tag{2.6}$$

where $\mathcal{C}$ is a dimension-full parameter which is discussed in appendix B.1. The integration over the $a_\mu$ and the $M^\alpha$ zero mode is interpreted as the superspace integration. Hence the stringy instanton contribution to the superpotential is given by

$$W_{\text{inst}} \sim \int d\{\lambda^{\dot{\alpha}}, D, \omega_{AA}, \bar{\omega}_{AA}, \mu_{AA}, \bar{\mu}_{AA}, \bar{\mu}_{Ab}, \mu_{cA}\}\, e^{-S_{\text{inst}}} \tag{2.7}$$

The bosonic integration is discussed in appendix B.1. As for the fermionic integration

$$W_{\text{inst}} \sim \int d\lambda^{\dot{\alpha}} d\bar{\mu}_{AA}\, d\mu_{AA} \prod_{b,c}(d\bar{\mu}_{Ab})^{N_b}\,(d\mu_{cA})^{N_c} e^{-S_{\text{inst}}} \tag{2.8}$$

the integral on $\lambda^{\dot{\alpha}}$ can be performed as in [8] using the $S_1$ part of the instanton action and it gives the ADHM fermionic constraints. It also saturates the fermionic integration on $\bar{\mu}_{AA}$ and $\mu_{AA}$. We end up with the integral

$$W_{\text{inst}} \sim \int \prod_{b,c}(d\bar{\mu}_{Ab})^{N_b}\,(d\mu_{cA})^{N_c} e^{-S_W} \tag{2.9}$$

and this fermionic integration gives

$$W_{\text{inst}} \sim \det\left(\frac{\partial W}{\partial(\Phi_{cA}\Phi_{Ab})}\right) \equiv \det\left(\mathcal{M}\right) \tag{2.10}$$

Notice that $\mathcal{M}$ is a a square matrix from the anomaly free condition for the node $A$, which is $\sum_b N_b = \sum_c N_c$.

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[1]This is necessary in order to take into account the contribution to this expression for non abelian superpotential and for superpotentials with terms involving more than 3 fields.

## 3. Discussion on the equivalence

The contribution generated by a stringy instanton on an SU(1) gauge node is here obtained from the strong dynamics of the gauge theory. This equivalence follows from the involutive property of the (limiting case) of Seiberg duality (i.e. the case with $N_f = N_c + 1$ for unitary gauge groups).

We consider the previous toric quiver gauge theory with a SU(1) gauge group labeled by $A$, with $N_f$ flavours spread on the nodes connected to the SU(1) one. The part of the superpotential involving the fields charged under the gauge group $A$ is a generic holomorphic function

$$W = W_0(\Phi_{cA}\Phi_{Ab}, X_{bc}^{(p)}) \tag{3.1}$$

where the $\Phi$ fields are bifundamentals charged under the SU(1). The $X_{bc}^{(p)}$ are fields or products of fields charged under the gauge groups connected to $A$ in the quiver. In section 2 we showed that a stringy instanton on node $A$ gives a contribution to the superpotential of the form

$$W_{\text{inst}} \sim \det \frac{\partial W_0}{\partial(\Phi_{cA}\Phi_{Ab})} \tag{3.2}$$

We now perform two consecutive Seiberg dualities on the node $A$ and compare the resulting theory with the original one. The first step is a formal Seiberg duality for the gauge group SU(1). This gives a SU($\tilde{N} = N_f - 1$) gauge theory with $N_f$ flavours, and superpotential

$$W_{\text{dual}} = W_0(M_{cb}, X_{bc}^{(p)}) + M_{cb}q_{bA}q_{Ac} \tag{3.3}$$

where $M_{cb} = \Phi_{cA}\Phi_{Ab}$ and $q_{bA}$ and $q_{Ac}$ are the dual quarks.

The next step is another duality on the node $A$. Since $N_f = \tilde{N} + 1$, the dual gauge group is SU(1), and the superpotential is

$$W_{\text{eff}} = W_0(M_{cb}, X_{bc}^{(p)}) + M_{cb}N_{bc} - N_{bc}b_{cA}b_{Ab} + \det N_{bc} \tag{3.4}$$

where $N_{bc} = q_{bA}q_{Ac}$, the $b$ are baryons, and we have changed the sign of the interaction term as in [21]. The last two terms implement the classical constraint on the moduli space [20].

For the involution to hold, this theory should coincide with the original one, after integrating out the massive mesons $M_{cb}$, $N_{bc}$. The equations of motions of the fields $N_{bc}$ give

$$b_{cA}b_{Ab} = M_{cb} = \Phi_{cA}\Phi_{Ab} \tag{3.5}$$

Hence we identify the baryons $b$ with the original fields $\Phi$. The equation of motion of the meson $M_{cb}$ implies that

$$N_{bc} \sim \frac{\partial W_0}{\partial M_{cb}} = \frac{\partial W_0}{\partial(\Phi_{cA}\Phi_{Ab})} \tag{3.6}$$

so we recover in (3.4) the original theory (3.1) and also the stringy instanton contribution (3.2), i.e. the determinant term. This proves the correspondence. We conclude that the involution of the Seiberg duality in the limiting case provides a gauge theory explanation of the stringy instanton contribution.
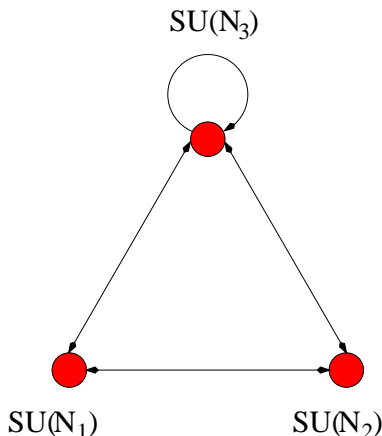
**Figure 2:** $L^{121}$ quiver gauge theory.

## 4. Examples

In this section we exhibit two examples of the correspondence: a non chiral theory, the $L^{121}$ quiver gauge theory, and the $dP_1$ chiral theory.

We begin with a theory where there is a node with $N_f = N_c + 1$, we then consider strong dynamics for that node and we study the low energy theory, performing a Seiberg duality in the limiting case, obtaining a non trivial contribution to the superpotential. The same contribution is obtained analyzing directly the low energy theory and taking into account the stringy instanton effect on the dualized node, an SU(1) node.

### 4.1 Non chiral example: $L^{121}$

The superpotential is

$$W = -X_{33}Q_{31}Q_{13} + X_{33}Q_{32}Q_{23} + Q_{21}Q_{13}Q_{31}Q_{12} - Q_{32}Q_{21}Q_{12}Q_{23} \qquad (4.1)$$

We choose the assignment of ranks for the gauge groups such that

$$N_2 + N_3 = N_1 + 1 \qquad (4.2)$$

We now consider strong dynamics for the node 1. This node has $N_f = N_c + 1$. The low energy can be analyzed performing a limiting case of Seiberg duality. The magnetic gauge group is SU(1), and the magnetic quarks are identified with the baryons of the electric description. The resulting theory has superpotential

$$W = -X_{33}M_{33} + X_{33}Q_{32}Q_{23} + M_{23}M_{32} - M_{22}Q_{23}Q_{32}$$
$$+ M_{33}q_{31}q_{13} + M_{22}q_{21}q_{12} - M_{23}q_{31}q_{12} - M_{32}q_{21}q_{13} + \det\begin{pmatrix} M_{22} & -M_{23} \\ -M_{32} & M_{33} \end{pmatrix} (4.3)$$

We have added the determinant contribution in order to correctly implement the classical constraint on the moduli space. Integrating out the massive fields, we obtain the quiver in
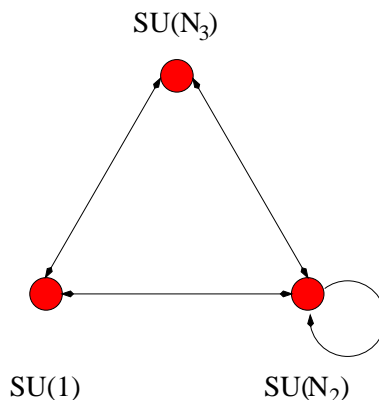
**Figure 3:** $L^{121}$ after dualizing node 1.

figure 3 with the following superpotential

$$W = -q_{21}q_{13}q_{31}q_{12} + Q_{23}q_{31}q_{13}Q_{32} + q_{12}M_{22}q_{21} - Q_{32}M_{22}Q_{23} + \det \begin{pmatrix} M_{22} & -q_{21}q_{13} \\ -q_{31}q_{12} & Q_{32}Q_{23} \end{pmatrix}$$
(4.4)

where there is an extra determinant term with respect to the usual SPP superpotential. The theory of figure 3 has an SU(1) gauge group. Strong dynamics effects from the theory one step backward in Seiberg duality have produced a non trivial superpotential contribution, i.e. the determinant term in (4.4). We show here that the same term is generated by a stringy instanton in the theory of figure 3.

The instantonic action for the $D$-instantons in the $SPP$ has been constructed in [18]:

$$\begin{aligned} S_{\text{inst}} = \ & i(\bar{\mu}_{11}\omega_{\dot{\alpha}11} + \bar{\omega}_{\dot{\alpha}11}\mu_{11})\lambda^{\dot{\alpha}} - iD^c(\bar{\omega}_{\dot{\alpha}11}(\tau^c)^{\dot{\alpha}}_{\dot{\beta}}\omega^{\dot{\beta}}_{11}) \\ & + \frac{1}{2}\bar{\omega}_{\dot{\alpha}11}\left(q_{12}q^{\dagger}_{12} + q^{\dagger}_{21}q_{21} + q_{13}q^{\dagger}_{13} + q^{\dagger}_{13}q_{31}\right)\omega_{\dot{\alpha}11} \\ & + \frac{i}{2}\left(\bar{\mu}_{12}q^{\dagger}_{12}\mu_{11} + \bar{\mu}_{13}q^{\dagger}_{13}\mu_{11} - \bar{\mu}_{11}q^{\dagger}_{21}\mu_{21} - \bar{\mu}_{11}q^{\dagger}_{31}\mu_{31}\right) \\ & - \frac{i}{2}\left(\bar{\mu}_{12}M_{22}\mu_{21} - \bar{\mu}_{12}q_{21}q_{13}\mu_{31} - \bar{\mu}_{13}q_{31}q_{12}\mu_{21} + \bar{\mu}_{13}Q_{32}Q_{23}\mu_{31}\right) \end{aligned}$$
(4.5)

The corresponding quiver is given in figure 4. The solid lines are the chiral superfields. The dashed lines are the fermionic zero modes connecting the instanton and the other $D$-branes in the theory. In order to compute the instanton contribution to the effective superpotential we have to integrate over the fermionic and bosonic zero mode that couple among them and with the chiral superfields. The $a_\mu$ and $M^\alpha$ zero modes are the superspace coordinates. The integral over the $\lambda^{\dot{\alpha}}$ and $D^c$ can be done using the first line in (4.5), and they give the two fermionic and the three fermionic ADHM constraints. The other bosonic integration has been shown in appendix B.1 to give only a constant. The fermionic ADHM constraints are

$$\begin{aligned} \delta\left(\bar{\mu}_{11}\omega_{\dot{1}11} + \bar{\omega}_{\dot{1}11}\mu_{11}\right)\delta\left(\bar{\mu}_{11}\omega_{\dot{2}11} + \bar{\omega}_{\dot{2}11}\mu_{11}\right) &= \left(\bar{\mu}_{11}\omega_{\dot{1}11} + \bar{\omega}_{\dot{1}11}\mu_{11}\right)\left(\bar{\mu}_{11}\omega_{\dot{2}11} + \bar{\omega}_{\dot{2}11}\mu_{11}\right) \\ &= \bar{\mu}_{11}\left(\omega_{\dot{1}11}\bar{\omega}_{\dot{2}11} - \omega_{\dot{2}11}\bar{\omega}_{\dot{1}11}\right)\mu_{11} \end{aligned}$$
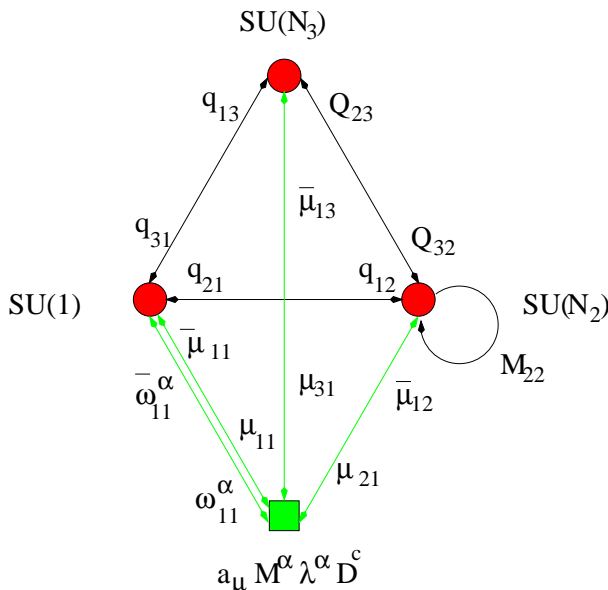(4.6)

**Figure 4:** $L^{121}$ with instanton on node $N_1 = 1$.

This term saturates also the integrations over the zero modes $\mu_{11}$ and $\bar{\mu}_{11}$.

We are left with the following fermionic integral

$$W_{\text{inst}} \sim \int d^{N_2}\bar{\mu}_{12}d^{N_2}\mu_{21}d^{N_3}\bar{\mu}_{13}d^{N_3}\mu_{31}e^{-S_{\text{inst}}} \tag{4.7}$$

The relevant part of the action for this integral is the last line in (4.5). It can be rearranged as

$$S_{\text{inst}} = \cdots - \frac{i}{2}\begin{pmatrix} \bar{\mu}_{12} & \bar{\mu}_{13} \end{pmatrix}\begin{pmatrix} M_{22} & -q_{21}q_{13} \\ -q_{31}q_{12} & Q_{32}Q_{23} \end{pmatrix}\begin{pmatrix} \mu_{21} \\ \mu_{31} \end{pmatrix} \tag{4.8}$$

and the fermionic integration (4.7) gives the contribution

$$W_{\text{inst}} \sim \det\begin{pmatrix} M_{22} & -q_{21}q_{13} \\ -q_{31}q_{12} & Q_{32}Q_{23} \end{pmatrix} \tag{4.9}$$

This is exactly the same determinant contribution we have obtained in (4.4). The correspondence between the superpotential terms holds. Indeed, adding (4.9) to the tree level superpotential for the quiver in figure 3, we exactly recover (4.4).

### 4.2 $dP_1$

Here we study the the chiral $dP_1$ toric quiver gauge theory. The quiver of the theory is in figure 5.a . The superpotential is

$$W = \epsilon_{\alpha\beta}X_{23}^\alpha X_{34}^\beta X_{42} + \epsilon_{\alpha\beta}X_{34}^\alpha X_{41}^\beta X_{13} - \epsilon_{\alpha\beta}X_{12}X_{23}^\alpha X_{34}^3 X_{41}^\beta \tag{4.10}$$

We choose the ranks to be

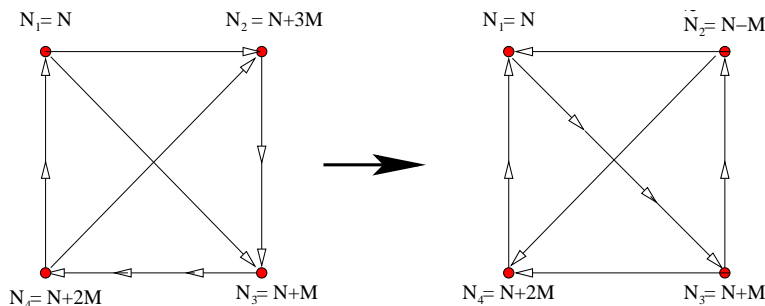$$N_1 = N \qquad N_2 = N + 3M \qquad N_3 = N + M \qquad N_4 = N + 2M$$

**Figure 5:** Quivers representing the two dual phases studied for $dP_1$

and consider strong dynamics for the node 2. The dual degrees of freedom are the dual quarks $(b^\alpha_{32}, b_{21}, b_{24})$ and the mesons $(M^\alpha_{13}, M^\alpha_{43})$. The resulting quiver is in figure 5.b and the superpotential, after integrating out the massive matter, is

$$W = \epsilon_{\alpha\beta} b^\alpha_{32} b_{24} X^\beta_{41} X_{13} + \epsilon_{\alpha\beta} M^\alpha_{13} b^\beta_{32} b_{21} - \epsilon_{\alpha\beta} M^\alpha_{13} X^3_{34} X^\beta_{41} \tag{4.11}$$

We used the equations of motion of the massive fields $(M^\alpha_{43}, X^\alpha_{34})$ that fix

$$M^1_{43} = X^1_{41} X_{13} \qquad M^2_{43} = X^2_{41} X_{13} \tag{4.12}$$

Choosing $N = M + 1$ we are in the limiting case of Seiberg duality, where the dualized magnetic gauge group is SU(1). The classical constraint on the moduli space in this case is implemented adding to the superpotential (4.11) a determinant term

$$\Delta W = \det \mathcal{M} = \det \begin{pmatrix} M^1_{13} & -M^2_{13} \\ -M^1_{43} & M^2_{43} \end{pmatrix} = \det \begin{pmatrix} M^1_{13} & -M^2_{13} \\ -X^1_{41} X_{13} & X^2_{41} X_{13} \end{pmatrix} \tag{4.13}$$

where we used the equation of motions to express it as a function of the fields of the effective theory.

We now recover the same contribution as a stringy instanton effect in the magnetic theory, the one in figure 5.b. We place a stringy instanton on the SU(1) node. The saturation of the zero modes proceed as usual and we are left with the following integral

$$W_{\text{inst}} \sim \int (d\bar{\mu}_{24})^{N_4} (d\bar{\mu}_{21})^{N_1} (d\bar{\mu}^1_{32})^{N_2} (d\bar{\mu}^2_{32})^{N_2} e^{-S_{\text{inst}}} \tag{4.14}$$

The relevant part of the instantonic action is (2.5) and can be deduced from the superpotential (4.11) to be

$$S_{\text{inst}} \supset \epsilon_{\alpha\beta} \bar{\mu}_{24} X^\beta_{41} X_{13} \mu^\alpha_{32} + \epsilon_{\alpha\beta} \bar{\mu}_{21} M^\alpha_{13} \mu^\beta_{32} = \begin{pmatrix} \bar{\mu}_{21} & \bar{\mu}_{24} \end{pmatrix} \begin{pmatrix} M^1_{13} & -M^2_{13} \\ -X^1_{41} X_{13} & X^2_{41} X_{13} \end{pmatrix} \begin{pmatrix} \mu^2_{32} \\ \mu^1_{32} \end{pmatrix} \tag{4.15}$$

Performing the fermionic integrals we then find that

$$W_{\text{inst}} \sim \det \begin{pmatrix} M^1_{13} & -M^2_{13} \\ -X^1_{41} X_{13} & X^2_{41} X_{13} \end{pmatrix} \tag{4.16}$$

that is $W_{\text{inst}} = \Delta W$ as claimed. The stringy instanton contribution has been exactly mapped to the strong dynamics effect.
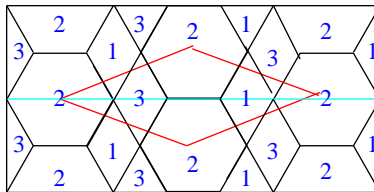
**Figure 6:** Dimer model for the fixed line orientifold of the SPP.

## 5. Orthogonal and symplectic gauge groups

In this section we generalize the correspondence to orthogonal and symplectic gauge groups. Quiver gauge theories with these groups can be obtained from unitary gauge groups applying orientifold projections. Since we consider toric quiver gauge theories, we use the technology developed in [22] to perform orientifold projections on dimer models [23].

The $O$-plane projects out some degrees of freedom both in the gauge sector and in the instanton sector. As a consequence the number of bosonic and fermionic zero modes and the corresponding ADHM constraints are different [24].

The stringy instanton contribution to the superpotential for symplectic and orthogonal gauge groups has been studied in [4, 5, 7, 12]. Non trivial contributions, in analogy with the SU(1) case, arise for stringy instantons of SP(0) and SO(3) gauge groups. The auxiliary instantonic groups are in these cases $O(1)$ and SP(2), respectively.[2]

The relation between stringy instantons and strong dynamics effects of the gauge theory holds also in these cases. Electric magnetic dualities have been studied in [20, 25, 26] for symplectic and orthogonal gauge groups. For these groups there exist limiting cases of the duality, as the $N_f = N_c + 1$ case for SU($N_c$). They are respectively the $N_f = N_c + 4$ for SP($N_c$) and $N_f = N_c - 1$ for SO($N_c$) gauge groups. For unitary groups the dual description is a SU(1) gauge theory. For symplectic and orthogonal gauge groups the dual descriptions are SP(0) and SO(3), respectively. Indeed they are the configuration where stringy instanton effects add to the superpotential.

We now point out the agreement between stringy instanton and gauge theory analysis with some example.

### 5.1 The orthogonal case

In this subsection we study an orientifold projection of the SPP. We choose an orientifold from the dimer with a fixed line, where the unit cell of the dimer has a rhombus geometry (see figure 6). The orientifold charge for the fixed line is chosen positive. In this case all the unitary groups SU($N_i$) become orthogonal SO($N_i$) groups. Half of the bifundamentals survive the projection, and they become

$$q_{i,j} = (\square_i, \square_j) \tag{5.1}$$

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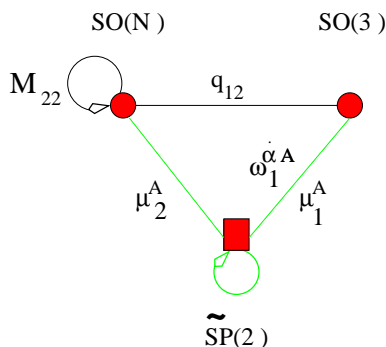[2]In our convention SP(2) $\simeq$ SU(2).

**Figure 7:** Zero modes (in green) for the $\tilde{SP}(2)$ instanton placed on an SO(3) gauge group.

The adjoint field $M_{22}$ is projected to a symmetric representation. We then choose the number of fractional branes for each group such that

$$N_1 = N \quad N_2 = 3 \quad N_3 = 0 \tag{5.2}$$

This theory is described by the superpotential

$$W = q_{12}^T q_{12} M_{22} \tag{5.3}$$

We add D-brane instantons on the SO(3) node. In the ADHM construction of SO($N$) $\mathcal{N} = 4$ SYM the instantonic auxiliary group [24] is SP($k$). In this case the counting of the zero modes tells that stringy instanton contributes to the superpotential if the auxiliary group is SP(2). The orientifolded quiver gauge theory with the instanton and the relative zero modes are shown in the figure 7. The action for the zero modes is

$$S_{\text{inst}} = i\lambda_{\dot{\alpha}}^i \left( w_1^{\dot{\alpha}a} \sigma_{ab}^i \mu_1^b \right) + i\lambda_{\dot{\alpha}}'^i \left( \bar{w}_1^{\dot{\alpha}a} \sigma_{ab}^i \mu_1^b \right) - iD_i^k \left( w_1^{\dot{\alpha}a} \sigma_{\dot{\alpha}\dot{\beta}}^k \sigma_{ab}^i \bar{w}_1^{\dot{\beta}b} \right)$$
$$+ \frac{1}{2} \left( w_1^{\dot{\alpha}a} \left( q_{12} q_{12}^\dagger \right) \bar{w}_1^{\dot{\alpha}a} + i\mu_2^a q_{12}^\dagger \mu_1^b \epsilon_{ab} - i\mu_2^a \epsilon_{ab} \mu_2^{T^b} M_{22} \right) \tag{5.4}$$

where $a$ and $b$ are SP(2) indexes. Imposing the reality conditions we find six independent $\lambda_{\dot{\alpha}}^i, \lambda_{\dot{\alpha}}'^i$ and four $M_\alpha$ zero modes. The $a_\mu$, in the adjoint of SP(2), are symplectic anti-symmetric matrices. This representation has dimension 1, which implies that there are four zero modes from $a_\mu$. The $D_c$ are nine while there are twelve independent $w_1^{\dot{\alpha}a}, \bar{w}_1^{\dot{\alpha}a}$ bosonic spinors. There are six fermionic $\mu_1^a$ fields connecting the gauge group SO(3) with the auxiliary instantonic group SP(2). The sector connecting the SP(2) instanton with the flavor group gives $2N_f$ fermionic zero modes $\mu_2^a$. We can now perform the integration over the fermionic and bosonic zero modes to obtain the instanton contribution. The $(a_\mu, M_\alpha)$ zero modes are as usual interpreted as superspace coordinates, giving the superpotential contribution

$$W_{\text{inst}} = \mathcal{C} \int d\{\lambda, \lambda', D, \omega_1, \bar{\omega}_1, \mu_1, \mu_2\} e^{-S_{\text{inst}}} \tag{5.5}$$

We discuss in the appendix B.2 the bosonic integration and the dimension-full constant $\mathcal{C}$. We only quote here the nine ADHM bosonic constraints obtained integrating over the $D_i^k$

$$\delta^{(9)} \left( w_c^{\dot{\alpha}a} \sigma_{\dot{\alpha}\dot{\beta}}^k \sigma_{ab}^i \bar{w}_c^{\dot{\beta}b} \right) \tag{5.6}$$

Now we focus on the fermionic integration. The integration over the $\lambda^i_\alpha, \lambda'^i_{\dot\alpha}$ fermionic zero modes can be done using the first two terms in (5.4) and it gives the six ADHM fermionic constraints

$$\delta^{(3)}(\omega^a_c \sigma^i_{ab} \mu^b_c)\delta^{(3)}(\bar\omega^a_c \sigma^i_{ab} \mu^b_c) \tag{5.7}$$

This saturate also the fermionic integration on $\mu^a_1$ in (5.5). We are left with the fermionic integral

$$W_{\text{inst}} \sim \int [d\mu^a_2]\, e^{-S_{\text{inst}}} \tag{5.8}$$

The integration is done expanding the relevant part of the action in the exponent

$$S_{\text{inst}} \supset \mu^1_2 \mu^{T\,2}_2 M_{22} - \mu^2_2 \mu^{T\,1}_2 M_{22} = \left( \mu^1_2\ \mu^{T\,2}_2 \right) \begin{pmatrix} 0 & M_{22} \\ -M_{22} & 0 \end{pmatrix} \begin{pmatrix} \mu^1_2 \\ \mu^{T\,2}_2 \end{pmatrix} \tag{5.9}$$

The gaussian integration gives the contribution

$$W_{\text{inst}} \sim \text{Pf} \begin{pmatrix} 0 & M_{22} \\ -M_{22} & 0 \end{pmatrix} = \det M_{22} \tag{5.10}$$

This last equality holds since $M$ is a symmetric matrix. In appendix B.2 we show, using dimensional analysis, that the bosonic integral is adimensional. This means that it is independent from the physical fields and it gives only a constant contribution. We conclude that (5.10) is the SP(2) stringy instanton contribution on the SO(3) node.

We now argue that the same relationship between stringy instanton and strong dynamics that holds in the case of unitary groups is valid also in this situation.

Once again we exploit the involutive property of Seiberg duality. We thus perform two consecutive Seiberg duality, recovering in the end the starting theory. The first one is a formal Seiberg duality on the SO(3) node with $N$ flavours, and we obtain the theory one step backwards. This gives an SO($\tilde N = N_f - N_c + 4 = N + 1$) gauge group with $N$ flavor. The superpotential of this theory is

$$W = Q^T_{12} Q_{12} N_{22} + N_{22} M_{22} \tag{5.11}$$

Integrating out the massive field this superpotential vanishes. We perform then another Seiberg duality. Since for this theory $N_f = N_c - 1$ we are in the limiting case of Seiberg duality for orthogonal gauge groups. The dual gauge group is SO($N_f - N_c + 4 = 3$) and the superpotential

$$W = q^T_{12} q_{12} M_{22} + \det M_{22} \tag{5.12}$$

where we have added the determinant to take into account the classical constraint on the moduli space.

We have thus recovered in (5.12) the starting superpotential (5.3) and the stringy instanton contribution (5.10). So also for orthogonal gauge group we have mapped the stringy instanton contribution in strong dynamics effects.
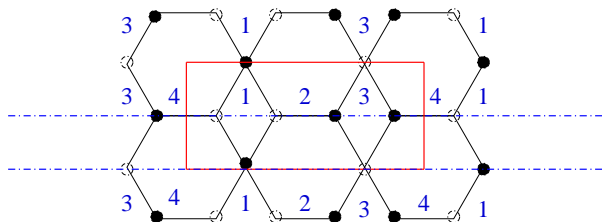
**Figure 8:** Dimer for the orientifold of the double conifold. The dashed blue lines represents the orientifold fixed lines.

## 5.2 The symplectic case

We first consider symplectic SQCD. We take $SP(0)$ as the gauge group, with $SP(N)$ flavours. There is a meson $M$ in the antisymmetric representation and there is no superpotential.

It has been shown that a stringy instanton on the $SP(0)$ gauge group gives a non trivial contribution to the superpotential. In the ADHM construction the instantonic auxiliary group for a symplectic gauge group is $O(k)$. The non perturbative contribution is obtained if the instantonic number is $k = 1$, with auxiliary group $O(1)$. There are no fermionic and bosonic ADHM constraint, no $w$, $D$ and $\lambda$ fields. There are two $M_\alpha$ and four $a_\mu$ which are interpreted as the superspace coordinates. The instantonic action is given by the interaction of the meson with the fermionic zero modes $\mu$ connecting the $O(1)$ instanton and the flavor group

$$S = -\frac{i}{2}\mu M \mu^T \tag{5.13}$$

The superpotential contribution is obtained integrating over the $\mu$ fermionic zero modes

$$W_{\text{inst}} \sim \int d[\mu] e^{-S} = \text{Pf} M \tag{5.14}$$

Also for symplectic gauge groups we relate this contribution to strong dynamics effects. Through a formal electric magnetic duality on the $SP(0)$ node we obtain the dual theory. It is an $SP(\tilde{N} = N_f - N_c - 4 = N - 4)$ gauge group with $N$ flavours and no mesons. We then perform another duality obtaining a $SP(N_f - N_c - 4 = N - \tilde{N} - 4 = 0)$ gauge group, where the only degree of freedom is the meson $M$. This is the starting theory. However, since we are in the limiting case of Seiberg duality for symplectic gauge group, we also obtain the following contribution to the superpotential

$$W_{\text{eff}} = \text{Pf} M \tag{5.15}$$

which implement the classical constraints on the moduli space. The equivalence between (5.14) and (5.15) shows that, also for symplectic gauge groups, the strong dynamic effect coincides with the stringy instanton contribution to the superpotential.

### 5.2.1 Example: orientifold of the double conifold

In this subsection we give an example of the correspondence using an orientifold of the double conifolds. The dimer model is represented in figure 8. The unit cell is delimited
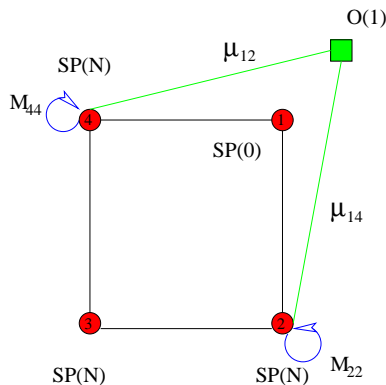
**Figure 9:** Stringy instanton on the orientifolded double conifold. The green lines represent the fermionic zero modes in the instantonic action.

by the red lines. The projection is done by the two independent dashed blue fixed lines. We choose their orientifold charge to be negative. This implies that all the groups are symplectic. The bifundamentals fields are in the $(\square_i, \square_j)$ representation of the $SP(2N_i) \times SP(2N_j)$ gauge groups. The fields in the adjoint representation of the $SU(N_i)$ gauge groups are now in the antisymmetric representation of the $SP(2N_i)$ groups. The rank of the first group $SP(2N_1)$ is chosen to be zero. The choice of the others ranks is free. Here, for simplicity, we choose the same rank $N$ for all of them. The superpotential for this theory is

$$W = M_{22} \cdot Q_{23} \cdot Q_{23} - Q_{23} \cdot Q_{23} \cdot Q_{34} \cdot Q_{34} + M_{44} \cdot Q_{34} \cdot Q_{34} \tag{5.16}$$

where the $\cdot$ represent the symplectic products.

We add a stringy instanton on the $SP(0)$ node and we study its contribution to the superpotential. The zero modes are shown in figure 9. The instantonic action is

$$S_{\text{inst}} = -\frac{i}{2}\mu_{12}M_{22}\mu_{12}^T - \frac{i}{2}\mu_{14}M_{44}\mu_{14}^T \tag{5.17}$$

The integration over the fermionic zero modes $\mu_{12}$ and $\mu_{14}$ gives a non perturbative contribution

$$W_{\text{inst}} \sim \int d[\mu_{12}]d[\mu_{14}]e^{-S_{\text{inst}}} = \text{Pf}M_{22}\text{Pf}M_{44} \tag{5.18}$$

to the superpotential (5.16).

The same result can be found from the gauge theory analysis. The theory one step backwards in Seiberg duality is obtained by a formal duality on the $SP(0)$ node. We get a $SP(2N-4)$ gauge group with $2N$ flavours. The superpotential of this dual theory is

$$\begin{aligned} W = Q_{12} \cdot Q_{12} \cdot Q_{23} \cdot Q_{23} - Q_{23} \cdot Q_{23} \cdot Q_{34} \cdot Q_{34} \\ + Q_{34} \cdot Q_{34} \cdot Q_{41} \cdot Q_{41} - Q_{41} \cdot Q_{41} \cdot Q_{12} \cdot Q_{12} \end{aligned} \tag{5.19}$$

We perform then another Seiberg duality to go back to the starting theory. We obtain a $SP(0)$ gauge group and superpotential

$$W = -M_{24} \cdot M_{42} + M_{22} \cdot Q_{23} \cdot Q_{23} - Q_{23} \cdot Q_{23} \cdot Q_{34} \cdot Q_{34} + M_{44} \cdot Q_{34} \cdot Q_{34} + \text{Pf}\begin{pmatrix} M_{22} & M_{24} \\ M_{42} & M_{44} \end{pmatrix} \tag{5.20}$$

The mesons are defined as $M_{ij} = q_{i\lambda_i} J_{\lambda_1 \lambda_2} q_{j,\lambda_2}$. Since we are in the case $N_f = N_c + 4$ we have added the non perturbative term to implement the classical constraint on the moduli space. Note that the antisymmetry of $J_{\lambda_1,\lambda_2}$ implies that $M_{42} = -M_{24}^T$. Integrating out the massive fields $(M_{24}, M_{42})$ we recover the starting theory (5.16) plus the non perturbative contribution

$$W_{\rm np} = {\rm Pf} M_{22} {\rm Pf} M_{44} \tag{5.21}$$

This is exactly the same contribution obtained from the stringy instanton computation (5.18).

## 6. Conclusions

In this paper we have considered stringy instantons in toric quiver gauge theories deriving from $D3/D(-1)$ systems. We have provided an interpretation for the stringy instanton contribution as a strong dynamics effect by analyzing the theory one step backwards in Seiberg duality, for the node where the instanton is located. Our result is valid for stringy instantons on SU(1), SP(0) and SO(3) nodes.[3]

There are interesting aspects we have not discussed here. The results we presented could be extended to non toric quiver gauge theories. Our analysis might also be useful in understanding the role played by stringy instantons in dynamical supersymmetry breaking in quiver gauge theories. Another issue would be the study of non-rigid instantons and multi-instantons effects in toric quiver gauge theories, and their relation to strong dynamics of the gauge theory. Finally[4] a similar correspondence should exist for instantonic higher $F$-term contributions in relation with strong dynamics leading to magnetic SU(0) gauge group, i.e. the $N_f = N_c$ case.

## Acknowledgments

## A. General result for U(1) instanton

In this appendix we compute the general contribution to the superpotential for a rigid U(1) instanton placed on a SU(1) node (denoted with $A$) in a toric quiver gauge theory, generalizing the result of section 2. The more general configuration includes the possibility of having more than one field with the same gauge group indexes, connected to the node $A$. We label these fields with an extra index $\alpha$ for outgoing arrow and $\beta$ for incoming

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[3]See [29] for related discussion in the context of matrix models.

[4]We thank A. M. Uranga for suggesting this to us.

arrow. Hence the fields connecting the node $A$ to the other gauge nodes are referred as $\Phi_{Ab}^{\alpha}$ or $\Phi_{cA}^{\beta}$. These extra indexes have to be inserted, and summed over, in all the formula of section 2, e.g. for the instantonic action. An important remark is that now the anomaly free condition for the node $A$ is

$$\sum_{b,\alpha} N_b = \sum_{c,\beta} N_c \tag{A.1}$$

The procedure for getting the superpotential contribution is as in section 2. The integration of the bosonic zero modes and of the fermionic zero mode $\lambda^{\dot{\alpha}}$ and $\mu_{AA}, \bar{\mu}_{AA}$, give the same result. We have to perform the following integral

$$W_{\text{inst}} \sim \int \prod_{b,\alpha,c,\beta} (d\bar{\mu}_{Ab}^{\alpha})^{N_b} (d\mu_{cA}^{\beta})^{N_c} e^{-S_W} \tag{A.2}$$

where now

$$S_W = -\frac{i}{2} \sum_{b,\alpha,c,\beta} \bar{\mu}_{Ab}^{\alpha} \frac{\partial W}{\partial(\Phi_{cA}^{\beta}\Phi_{Ab}^{\alpha})} \mu_{cA}^{\beta} \tag{A.3}$$

In order to compute this integral we can arrange the fermionic variable in vectors

$$\bar{\mu}_{AB} = (\mu_{Ab}^{\alpha}) \qquad \mu_{CA} = (\mu_{cA}^{\beta})^T \tag{A.4}$$

of dimension

$$B = 1,\ldots,\sum_{b,\alpha} N_b \qquad C = 1,\ldots,\sum_{c,\beta} N_c \tag{A.5}$$

and rewrite the instantonic action as

$$S_W = -\frac{i}{2} \bar{\mu}_{AB} \mathcal{M}_{BC} \mu_{CA} \tag{A.6}$$

where $\mathcal{M}$ is a matrix of dimension $\sum_{b,\alpha} N_b \times \sum_{c,\beta} N_c$, built taking derivatives of the superpotential

$$\mathcal{M} = \left( \frac{\partial W}{\partial(\Phi_{cA}^{\beta}\Phi_{Ab}^{\alpha})} \right) \tag{A.7}$$

$\mathcal{M}$ is a square matrix because of the anomaly free condition (A.1). The ordering of the fields in building $\mathcal{M}$ is irrelevant for the final contribution to the superpotential, which is a determinant. Indeed we can perform the fermionic integration and obtain the stringy instanton contribution

$$W_{\text{inst}} \sim \det \left( \frac{\partial W}{\partial(\Phi_{cA}^{\alpha}\Phi_{Ab}^{\beta})} \right) \tag{A.8}$$

## B. Bosonic integration

In this appendix we show, via dimensional arguments similar to [8], that the integration over the bosonic zero modes of the stringy instantons change the results of the fermionic integration only by a constant factor. We analyze the general bosonic integration for the U(1) and the SP(2) instanton. The $O(1)$ case is trivial since there are no bosonic zero modes to integrate over.

### B.1 The U(1)-instanton

In section 2 we have considered an SU(1) node $A$ in the quiver and label with index $b$ all the outgoing arrows, and with $c$ all the incoming arrows. We have then the collection of fields $\Phi_{Ab}$ and $\Phi_{cA}$.

We have seen that the contribution to the superpotential after fermionic integration, due to an instanton on node $A$ is given by the determinant of the squared matrix $\mathcal{M}$. The determinant of this matrix has mass dimension

$$[\det \mathcal{M}] = M_s^{(\sum_c N_c)} = M_s^{(\sum_b N_b)} \tag{B.1}$$

We can now compute the dimension of the measure factor for the general instanton computation of section 2

$$Z = \mathcal{C} \int d\{a_\mu, M, \lambda, D, \omega_{AA}, \bar{\omega}_{AA}, \mu_{AA}, \bar{\mu}_{AA}, \bar{\mu}_{Ab}, \mu_{cA}\} \, e^{-S_{\text{inst}}} \tag{B.2}$$

The dimension-full coefficient $\mathcal{C}$ is for the moment unknown. Using the usual standard dimensions we arrive at

$$[d\{a_\mu, M, \lambda, D, \omega_{AA}, \bar{\omega}_{AA}, \mu_{AA}, \bar{\mu}_{AA}, \bar{\mu}_{Ab}, \mu_{cA}\}] = M_s^{-n_a + \frac{1}{2}n_M - \frac{3}{2}n_\lambda + 2n_D - n_{\omega,\bar{\omega}} + \frac{1}{2}n_{\mu,\bar{\mu}}} \tag{B.3}$$

Since

$$n_a = 4 \quad n_M = n_\lambda = 2 \quad n_D = 3 \quad n_{\omega,\bar{\omega}} = 4N_A \quad n_{\mu\bar{\mu}} = 2N_A + \sum_b N_b + \sum_c N_c \tag{B.4}$$

we obtain

$$[d\{a_\mu, M, \lambda, D, \omega_{AA}, \bar{\omega}_{AA}, \mu_{AA}, \bar{\mu}_{AA}, \bar{\mu}_{Ab}, \mu_{cA}\}] = M_s^{-(3N_A - \frac{1}{2}(\sum_b N_b + \sum_c N_c))} = M_s^{-\beta_A} \tag{B.5}$$

where we have recognized the 1 loop beta function of the node $A$.

Now, since $Z$ in (B.2) should be adimensional we obtain that

$$\mathcal{C} = \Lambda^{\beta_A} \tag{B.6}$$

Hence we have

$$Z = \Lambda^{\beta_A} \int d\{a_\mu, M, \lambda, D, \omega_{AA}, \bar{\omega}_{AA}, \mu_{AA}, \bar{\mu}_{AA}, \bar{\mu}_{Ab}, \mu_{cA}\} \, e^{-S_{\text{inst}}} \tag{B.7}$$

Now, we expect that

$$Z = \int d^4x \, d^2\theta \, W_{\text{inst}} \tag{B.8}$$

and then

$$W_{\text{inst}} = \Lambda^{\beta_A} \int d\{\lambda, D, \omega_{AA}, \bar{\omega}_{AA}, \mu_{AA}, \bar{\mu}_{AA}, \bar{\mu}_{Ab}, \mu_{cA}\} \, e^{-S_{\text{inst}}} \tag{B.9}$$

Now, we have seen that the fermionic (plus the $D$) integrations give, when $N_A = 1$, the following

$$W_{\text{inst}} = \Lambda^{\beta_A} I_{\text{boson}} \det \mathcal{M} \tag{B.10}$$

where $\mathcal{M}$ is the meson built before and we still have to perform the bosonic integration $I_{\text{boson}}$, and show that it gives a numerical coefficient. Indeed the dimensional analysis gives

$$[W] = [\Lambda^{\beta_A}] + [I_{\text{boson}}] + [\det \mathcal{M}]\beta_A + [I_{\text{boson}}] + \left( \sum_b N_b \right) \tag{B.11}$$

$$= 3 - \frac{1}{2}\left( \sum_b N_b + \sum_c N_c \right) + [I_{\text{boson}}] + \left( \sum_b N_b \right) = 3 + [I_{\text{boson}}]$$

where we have used the anomaly free condition. In order to have a superpotential of dimension 3 we have to set $[I_{\text{boson}}] = 0$, i.e. a number.

### B.2 The $SP(2)$-instanton

We can easily repeat the analysis done in the previous section for the $SP(2)$ instanton on the $SO(3)$ gauge node. We denote with $A$ the $SO(3)$ gauge group where we place the instantons and label with index $b$ all the outgoing arrows, and with $c$ all the incoming arrows. In general the contribution to the superpotential after fermionic integration, due to $SP(2)$ instantons on node $A$ is given by a pfaffian of dimension

$$[\text{Pf}\mathcal{M}] = M_s^{(\sum_c N_c)} = M_s^{(\sum_b N_b)} \tag{B.12}$$

We can now compute the dimension of the instanton measure factor

$$Z = \mathcal{C} \int d\{a_\mu, M, \lambda, D, \omega_{AA}, \mu_{AA}, \bar{\mu}_{Ab}, \mu_{cA}\} \, e^{-S_{\text{inst}}} \tag{B.13}$$

The dimension-full coefficient $\mathcal{C}$ is up to now unknown. Using the usual dimensions we arrive at

$$[d\{a_\mu, M, \lambda, D, \omega_{AA}, \mu_{AA}, \bar{\mu}_{Ab}, \mu_{cA}\}] = M_s^{-n_a + \frac{1}{2}n_M - \frac{3}{2}n_\lambda + 2n_D - n_{\omega,\bar{\omega}} + \frac{1}{2}n_{\mu,\bar{\mu}}} \tag{B.14}$$

Now we have to remind that the auxiliary group for the instanton is $SP(2)$ and this gives different numbers of components respect to the $U(1)$ case, that is

$$n_a = 4 \quad n_M = 2, \quad n_\lambda = 6 \quad n_D = 9 \quad n_\omega = 4N_A \quad n_{\mu\bar{\mu}} = 2N_A + \sum_b N_b + \sum_c N_c \tag{B.15}$$

we obtain

$$[d\{a_\mu, M, \lambda, D, \omega_{AA}, \mu_{AA}, \bar{\mu}_{Ab}, \mu_{cA}\}] = M_s^{-(3N_A - 6 - \frac{1}{2}(\sum_b N_b + \sum_c N_c))} = M_s^{-\beta_A} \tag{B.16}$$

where we have recognized the 1 loop beta function of the $SO(N_A)$ node.

Now, since $Z$ in (B.13) should be adimensional we obtain that

$$\mathcal{C} = \Lambda^{\beta_A} \tag{B.17}$$

Hence we have

$$Z = \Lambda^{\beta_A} \int d\{a_\mu, M, \lambda, D, \omega_{AA}, \mu_{AA}, \bar{\mu}_{Ab}, \mu_{cA}\} \, e^{-S_{\text{inst}}} \tag{B.18}$$

Now, we expect that

$$Z = \int d^4 x d^2 \theta \, W_{\text{inst}} \tag{B.19}$$

and then

$$W_{\text{inst}} = \Lambda^{\beta_A} \int d\{\lambda, D, \omega_{AA}, \mu_{AA}, \bar{\mu}_{Ab}, \mu_{cA}\} \, e^{-S_{\text{inst}}} \tag{B.20}$$

Now, we have seen that the fermionic (plus the $D$) integrations give, when $N_A = 3$, the following

$$W_{\text{inst}} = \Lambda^{\beta_A} I_{\text{boson}} \text{Pf} \mathcal{M} \tag{B.21}$$

where $I_{\text{boson}}$ is the bosonic integration. The dimensional analysis told us that

$$[W] = [\Lambda^{\beta_A}] + [I_{\text{boson}}] + [\text{Pf}\mathcal{M}]\beta_A + [I_{\text{boson}}] + \left( \sum_b N_b \right) \tag{B.22}$$

$$= 3N_A - 6 - \frac{1}{2} \left( \sum_b N_b + \sum_c N_c \right) + [I_{\text{boson}}] + \left( \sum_b N_b \right) = 3N_A - 6 + [I_{\text{boson}}]$$

Since we have $N_A = 3$, in order to have a superpotential of dimension 3 we have to set $[I_{\text{boson}}] = 0$, i.e. a number.

## C. Relation with known models

In this appendix we show that there is no disagreement between the stringy instanton contributions of [8, 7] and our results.

### C.1 The $SU(1)$ theory

The theory studied in [8] is the $\mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orbifold. This is a quiver gauge theory with four gauge groups, described by the superpotential

$$W = \Phi_{12}\Phi_{23}\Phi_{31} - \Phi_{13}\Phi_{32}\Phi_{21} + \Phi_{13}\Phi_{34}\Phi_{41} - \Phi_{14}\Phi_{43}\Phi_{31}$$
$$+ \Phi_{23}\Phi_{34}\Phi_{42} - \Phi_{24}\Phi_{43}\Phi_{32} + \Phi_{12}\Phi_{24}\Phi_{41} - \Phi_{14}\Phi_{42}\Phi_{21} \tag{C.1}$$

The ranks of the groups are $(N_1, N_2, N_3, N_4) = (N_1, N_2, 1, 0)$. A stringy instanton placed on node $N_3$ contributes to the superpotential only if $N_1 = N_2$. In this case it has be shown that its contribution is

$$W_{\text{inst}} = \det \Phi_{12} \det \Phi_{21} \tag{C.2}$$

We now find the same result from gauge theory analysis. Dualizing the node 3 we find a theory with gauge group $SU(\tilde{N}_3 = N_1 + N_2 - 1)$ and superpotential

$$W = M_{11}Q_{13}Q_{31} - M_{22}Q_{23}Q_{32} \tag{C.3}$$

We then dualize again node 3. Since it is in the case $N_f = N_c + 1$, the dual theory has $SU(N_3) = SU(1)$ gauge group, and the superpotential is

$$W = M_{11}\Phi_{11} - M_{22}\Phi_{22} + \Phi_{11}\Phi_{13}\Phi_{31} - \Phi_{22}\Phi_{23}\Phi_{32} + \Phi_{12}\Phi_{23}\Phi_{31} - \Phi_{13}\Phi_{32}\Phi_{21} + \det \begin{pmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{pmatrix} \tag{C.4}$$
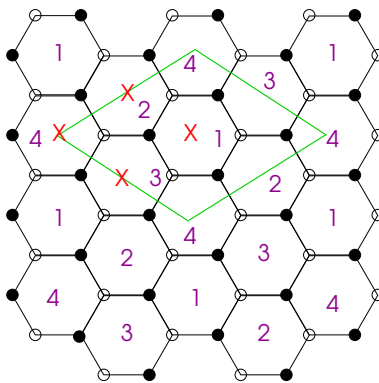
**Figure 10:** Unit cell and fixed point for $\mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$

After the integration of the massive field $M_{11}, M_{22}, \Phi_{11}, \Phi_{22}$, the superpotential is

$$W = \Phi_{12}\Phi_{23}\Phi_{31} - \Phi_{13}\Phi_{32}\Phi_{21} + \det \begin{pmatrix} 0 & \Phi_{12} \\ \Phi_{21} & 0 \end{pmatrix} \tag{C.5}$$

The first two terms are the same than (C.1). The det piece coincide with (C.2), as expected. Note that it vanishes if $N_1 \neq N_2$ as in the stringy instanton computation.

### C.2 The SP(0) theory

It is also possible to make an orientifold projection of the $\mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orbifold. A possible orientifold is described by the dimer in figure 10.

It is a fixed point projection. Since $N[W] = 8$, the total orientifold charge is positive. This condition can be imposed choosing all the single charge to be negative. All the groups are identified with themselves and they are all symplectic. All the fields are bifundamental in the $(\square_i, \square_j)$ of the $SP(N_i) \times SP(N_j)$ gauge groups. There is no superpotential for these fields. This is the same projection described in [7]. In that paper the ranks were $(N_1, N_2, N_3, N_4) = (N, N, 0, 0)$, and the stringy instanton was located on the third node. The stringy instanton contribution to the superpotential is given by

$$W_{\text{inst}} = \det \Phi_{12} \tag{C.6}$$

The same result can be found by the gauge theory analysis. The dual theory has rank $2N - 4$ for the third node, and superpotential

$$W = M_{11} \cdot Q_{13} \cdot Q_{13} - M_{22} \cdot Q_{23} \cdot Q_{32} \tag{C.7}$$

We then perform again electric magnetic duality on the third node. The gauge group becomes $SP(N_1 + N_2 - N_3 - 4) = SP(0)$, and the superpotential is

$$W = M_{11} \cdot \Phi_{11} - M_{22} \cdot \Phi_{22} + \text{Pf} \begin{pmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{pmatrix} \tag{C.8}$$

where all the blocks of the meson are in an antisymmetric representation of the flavor, and $\Phi_{21} = -\Phi_{12}^T$. Integrating out the massive fields the only non vanishing term of the superpotential is the non perturbative one

$$W = \text{Pf} \begin{pmatrix} 0 & \Phi_{12} \\ -\Phi_{12}^T & 0 \end{pmatrix} = \det \Phi_{12} \tag{C.9}$$

It is exactly the same than the stringy instanton contribution (C.6).

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